Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number

Candidate Number

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Model Solutions

Wednesday 16 May 2018 – Morning
Time: 2 hours

Paper Reference
8MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 5 8 3 4 6 A 0 1 4 8

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)dx = \frac{\frac{2}{3}x^4}{4} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + x + c$$

$$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$$

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2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

i) $x^2 - 8x + 17 = (x-4)^2 + 17 - 16$

$= (x-4)^2 + 1$

$(x-4)^2$ is always greater than 0

$\therefore x^2 - 8x + 17 > 0$ for all real values of x

ii) for $n=1$, $(1+3)^2 = 16$ $1^2 = 1$

$16 > 1$ $\therefore$ true for $n=1$

for $n=-3$ $(-3+3)^2 = 0$ $(-3)^2 = 9$

$0 < 9$ $\therefore$ false for $n=-3$

$\therefore$ statement is sometimes true

3. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

3a) $\overrightarrow{AB} = \begin{pmatrix} -5 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

$= -9i + 3j$

b) $|\overrightarrow{AB}| = \sqrt{(-9)^2 + 3^2}$

$= \sqrt{90}$

$= 3\sqrt{10}$

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4. The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

l_1 $\quad 4y - 3x = 10$

$y = \frac{3x+10}{4}$

$m = \frac{3}{4}$

l_2 $\quad m = \frac{8-(-1)}{-1-5}$

$= -\frac{3}{2}$

$\frac{3}{4} \times \left(-\frac{3}{2}\right) = -\frac{9}{8}$

$-\frac{9}{8} \neq -1$

gradients are not the same

$\therefore$ neither

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5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\dfrac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

a. using subtraction law for logs : the 2 in front should be the power of x

$\log_2 x^2 - \log_2 \sqrt{x} = 3$

$\log_2 \dfrac{x^2}{\sqrt{x}} = 3$

definition of a log : $x = 2^3$

$= 8$

b) $2\log_2 x - \log_2 \sqrt{x} = 3$

$\log_2 x^2 - \log_2 \sqrt{x} = 3(\log_2 2)$

$\log_2 \dfrac{x^2}{\sqrt{x}} = \log_2 2^3$

$\dfrac{x^2}{\sqrt{x}} = 8$

$x^{\frac{3}{2}} = 8$

$x = 8^{\frac{2}{3}}$

$= 4$

6.

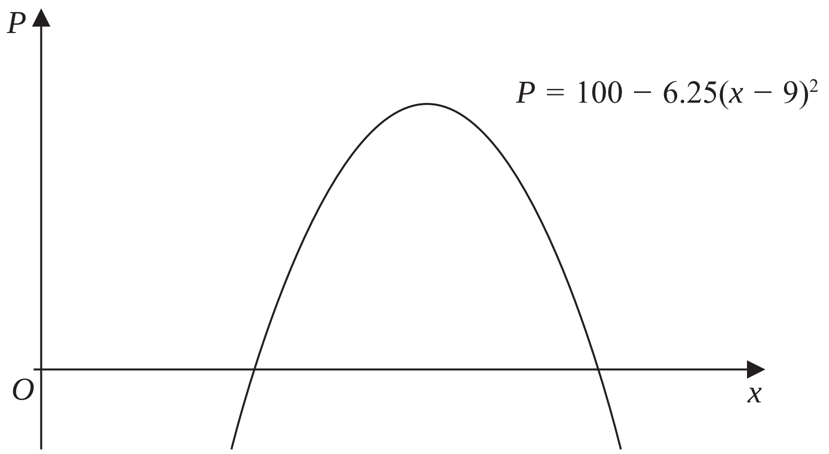

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

a) $P = 100 - 6.25(15-9)^2$

$= -125$

negative profit so company will make a loss

Question 6 continued

b) $P > 80$

$100 - 6.25(x-9)^2 > 80$

$(x-9)^2 > 3.2$

$x > 9 + \frac{4\sqrt{5}}{5}$, $x > 9 - \frac{4\sqrt{5}}{5}$

$x > 7.21$

$\therefore$ min price : $x = £7.22$

ci) $(x-9) = 0$

$P = 100$

Max. profit : £100 000

ii) $x - 9 = 0$

$x = 9$

Selling price : £9

7. In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

7a) $\frac{1}{2}(10)(5)\sin\theta = 15$

$\sin\theta = \frac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{16}{25}$

$\cos\theta = \pm\frac{4}{5}$

b) $BC^2 = 10^2 + 5^2 - 2(10)(5)\left(-\frac{4}{5}\right)$

$= 125 + 100\left(\frac{4}{5}\right)$

$= 205$

$BC = \sqrt{205}$

≈ 14.3 cm

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8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

ai) $\frac{dC}{dv} = 0$

$C = 1500v^{-1} + \frac{2}{11}v + 60$

$\frac{dC}{dv} = (-1)(1500v^{-2}) + \frac{2}{11}$

$= -\frac{1500}{v^2} + \frac{2}{11}$

$\frac{2}{11} - \frac{1500}{v^2} = 0$

$\frac{2}{11} = \frac{1500}{v^2}$

$2v^2 = 11(1500)$

$v = \sqrt{8250}$

$= 90.8 \text{ kmh}^{-1}$

ii) $C = \frac{1500}{90.8} + \frac{2(90.8)}{11} + 60$

$= 93.0289$

$\simeq £93.03$

b) $\frac{d^2C}{dv^2} = (-2)\frac{(-1500)}{v^3}$

$= \frac{3000}{v^3}$

$v = 90.8 \quad \frac{d^2C}{dv^2} = 0.004 > 0$

$\therefore$ minimum point

c) The speed throughout the whole journey cannot be kept constant.

9. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

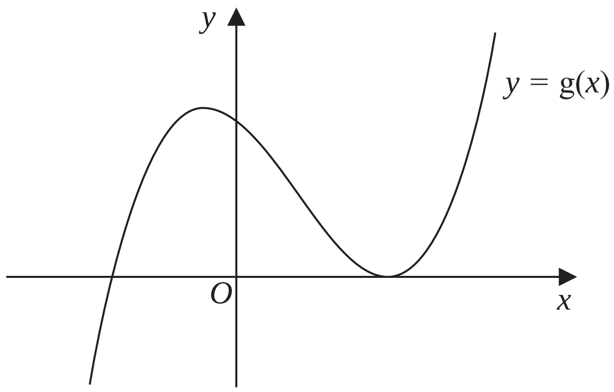

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

a) $(x+2) = 0 \quad x = -2$

$g(-2) = 4(-2)^3 - 12(-2)^2 - 15(-2) + 50$

$= 0$

$\therefore x+2$ is a factor

b)
$$
\begin{array}{r|l}
 & 4x^2 - 20x + 25 \\
x+2 & 4x^3 - 12x^2 - 15x + 50 \\
 & 4x^3 + 8x^2 \\
 & -20x^2 - 15x \\
 & -20x^2 - 40x \\
 & 25x + 50 \\
 & 25x + 50
\end{array}
$$

$g(x) = (x+2)(4x^2 - 20x + 25)$

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Question 9 continued

c i) $(x+2)(4x^2-20x+25)=0$

$x=-2$ $\quad (2x-5)^2=0$

$x=\pm 2.5$

$x<-2$ $\quad x=2.5$

ii) $x=-1$ $\quad x=1.25$

10. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$

gradient of chord $= 3x^2 + 3xh + h^2$

when $h \to 0$, $3xh \to 0$ $h^2 \to 0$

gradient of curve $= 3x^2$

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11. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

a) $\left(2-\frac{x}{16}\right)^9 = 2^9 + 9(2)^8\left(-\frac{x}{16}\right) + 36(2)^7\left(-\frac{x}{16}\right)^2 \ldots$

$= 512 - 144x + 18x^2, \ldots$

b) $a(512) = 128$

$a = \frac{1}{4}$

$b(512) + a(-144) = 36$

$512b = 36 + 144\left(\frac{1}{4}\right)$

$= 72$

$b = \frac{9}{64}$

12. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

12a) $4\cos\theta - 1 = 2\sin\theta\tan\theta$

$= 2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{2\sin^2\theta}{\cos\theta}$

$= \frac{2(1-\cos^2\theta)}{\cos\theta}$

$= \frac{2}{\cos\theta} - 2\cos\theta$

$6\cos\theta - 1 - \frac{2}{\cos\theta} = 0$

$\times \cos\theta$ $\quad 6\cos^2\theta - \cos\theta - 2 = 0$

b) $4\cos 3x - 1 = 2\sin x\tan 3x$

$6\cos^2(3x) - \cos 3x - 2 = 0$

$(3\cos 3x - 2)(2\cos 3x + 1) = 0$

$\cos 3x = \frac{2}{3}$ $\qquad \cos 3x = \frac{-1}{2}$

$3x = 48.19°$ $\qquad 3x = 120°, 240°$

$x = 16.1°$ $\qquad x = 40°, 80°$

13.

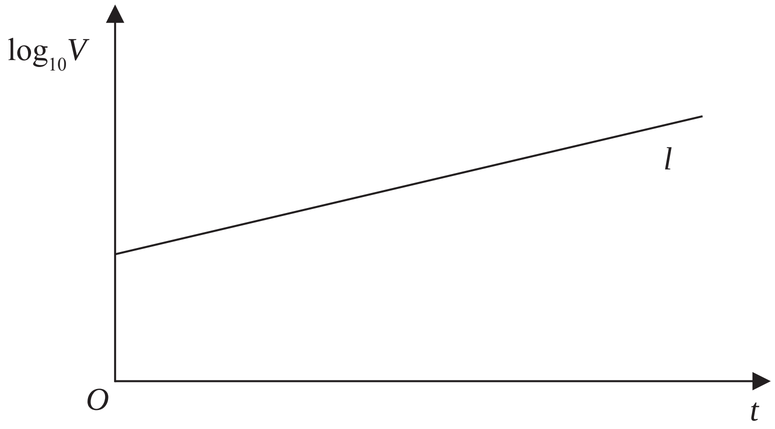


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

a) $p = 10^{4.8}$ $\quad$ $q = 10^{0.05}$

$= 63095.7$ $\quad$ $= 1.122018$

≈ 63100 $\quad$ ≈ 1.122

bi) value of painting on 1st January 1980

ii) The proportional increase of the value each year

c) $2010 - 1980 = 30$

$\log_{10} V = 0.05(30) + 4.8$

$V = 10^{6.3}$

$= 1995262$

$\approx$ £2000000

14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

ai) $(x-3)^2 + (y+5)^2 - 9 - 25 + 9 = 0$

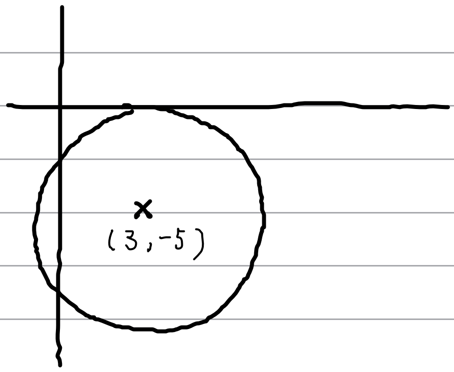
$(x-3)^2 + (y+5)^2 = 25$

centre : $(3, -5)$

ii) $r = \sqrt{25}$

$= 5\text{cm}$

b)



$y = kx$

$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$

$(1+k^2)x^2 + (10k-6)x + 9 = 0$

$a = 1+k^2 \quad b = 10k-6 \quad c = 9$

$b^2 - 4ac > 0$

$(10k-6)^2 - 4(1+k^2)(9) = 100k^2 - 120k + 36 - 36 - 36k^2$

$= 64k^2 - 120k$

$= k(8k-15)$

C.V. $k(8k-15) = 0$

$k = 0 \quad k = \frac{15}{8}$

$b^2 - 4ac > 0$ so $k < 0$, $k > \frac{15}{8}$

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15.

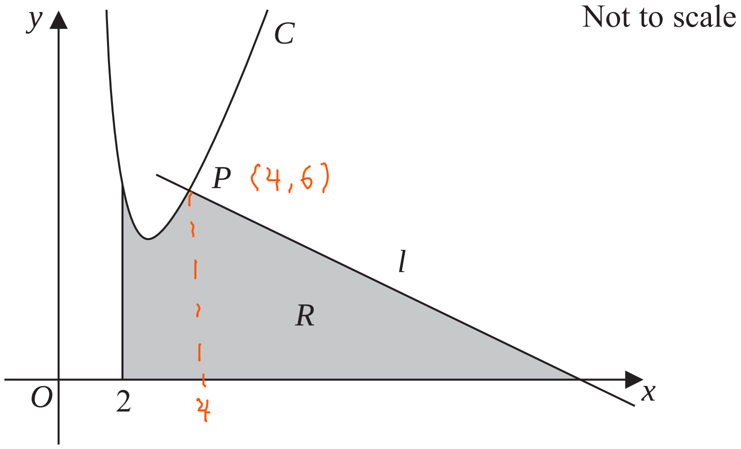

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

15) $\int_2^4 y\,dx = \left[\frac{-32}{x} + \frac{3x^2}{2} - 8x\right]_2^4$

$= -16 - (-26)$

$\boxed{= 10}$

$\frac{dy}{dx} = -2(32)x^{-3} + 3$

$x = 4 \quad \frac{dy}{dx} = 2$

gradient of $l = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$

$6 = -\frac{1}{2}(4) + c$

$c = 8$

$l: \quad y = -\frac{1}{2}x + 8$

when $y = 0$, $\quad 0 = -\frac{1}{2}x + 8$

$x = 16$

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Question 15 continued

$$\int_4^{16} -\frac{1}{2}x + 8 \, dx = \left[\frac{-\frac{1}{2}x^2}{2} + 8x \right]_4^{16}$$

$$= (64 - 28)$$

$$\boxed{= 36}$$

Total area $= 10 + 36$

$$\boxed{= 46}$$